Brackets & Equations

Review: Breaking Brackets

You already know how to break a single bracket and collect like terms:

e.g. i) $3(2x-5) \rightarrow 6x-15$ and ii) $-2(5y-3) \rightarrow -10y+6$

Recall the rules of signs:

+	×	+	→	+
+	×	–	→	–
–	×	+	→	–
–	×	–	→	+

We had the rule: **SAME SIGN PLUS**.

A **plus** times a **plus** makes a **plus**, and a **minus** times a **minus** makes a **plus** – different signs make a minus.

Review: Collecting like terms

We can deal with more complicated expressions as follows:

$3(5x-3)-2(4x+1) \rightarrow 15x-9-8x-2$ we can then collect like terms (tidy up) $\rightarrow 7x-11$

We can also tidy up expressions such as:

$x(2x+1)+3(4x-3) \rightarrow 2x^2+x+12x-9$ which will tidy up (simplify) to: $\rightarrow 2x^2+13x-9$

We can also tidy up expressions such as:

$$3a^2b+5ab+3ab^2-2ab-a^2b+2ab^2$$

noting that the like terms **have to have** the same powers (indices) $\rightarrow 2a^2b+3ab+5ab^2$

You should note that: *pq* is the same as *qp*. The order in which you multiply does not matter.

So: $3pq+2q^2-qp-p^2$ will simplify to: $\rightarrow 2pq+2q^2-p^2$

Review: Evaluating expressions

Expressions are simply rules indicating how to combine the numbers that the letters represent.

Examples: if $x=2,\quad y=-3,\quad z=5$

then i) $x+y \rightarrow 2+(-3) \rightarrow 2-3 \rightarrow -1$

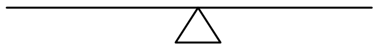
also ii) $\dfrac{x-y}{z} \rightarrow \dfrac{2-(-3)}{5} \rightarrow \dfrac{5}{5} \rightarrow 1$

and iii) $x^2-y^2 \rightarrow 2^2-(-3)^2 \rightarrow 4-(9) \rightarrow -5$

Brackets & Equations

Review: Solving equations

We can solve equations by thinking in terms of a balance.



Whatever you do to one side – you do to the other.

The aim is to get the variable (letter) on one side and the numbers on the other .

- You can add the same number to, or subtract the same number from both sides.
- You can multiply or divide both sides by the same number.
- You can even square each side or take the square root of each side.

e.g. $3x+5=17$

$3x=12$ *(subtracting 5 from each side)*

$x=4$ *(divide each side by 3)*

Collect the letters where there are the **MOST** of them, so you do not get negative letters.

e.g. $4x-3=9x+22$

$-3=5x+22$ *(subtracting 4x from each side)*

$-25=5x$ *(subtracting 22 from each side)*

$-5=x$ *(divide each side by 5)*

By putting together all the above, we can solve what appear to be quite complicated equations:

e.g. $3(2x-3)+4(3x-1)=2(x+5)+9$

$6x-9+12x-4=2x+10+9$ *(by breaking the brackets)*

$18x-13=2x+19$ *(by tidying up each side)*

$18x-2x=19+13$ *(take 2x from each side; add 13 to each side)*

$16x=32$ *(simplify each side)*

$x=2$ *(divide each side by 16)*

We can even deal with equations like this, by simply following the rules.

e.g. $x(2x+3)=2(x^2-3)$

$2x^2+3x=2x^2-6$ *(by breaking the brackets)*

$3x=-6$ *(subtracting $2x^2$ from each side)*

$x=-2$ *(divide each side by 3)*

Review: Solving inequalities

We solve inequalities by treating them exactly the same as we would an equation

However, you should take care

– if you **multiply** or **divide** by a **NEGATIVE** number, you **MUST** change the **direction** of the inequality sign.

e.g. $4x-3\geq 13$

$4x\geq 16$ *(adding 3 to each side)*

$x\geq 4$ *(divide each side by 4)*

e.g. $2-5x\leq 17$

$-5x\leq 15$ *(subtract 2 from each side)*

$x\geq -3$ *(divide each side by –5; note **change** in **direction** of inequality sign)*

Brackets & Equations

Breaking Pairs of Brackets – FOIL

We can break a single bracket – but what about two brackets multiplied together.

e.g. $(x+3)(x+4)$ First we need to imagine what this might represent.
It is two numbers multiplied together. It could be an area.

Imagine a garden of length: $x + 3$ and width $x + 4$ Then the above expression would represent the area.

	x	4
x		
3		

We could then work out the area of each section.

	x	4
x	x^2	$4x$
3	$3x$	12

This is also known as a **Multiplication** table

and by adding the individual areas we would get: $x^2+4x+3x+12$

so in fact; $(x+3)(x+4) \rightarrow x^2+4x+3x+12$ which would simplify to: $x^2+7x+12$

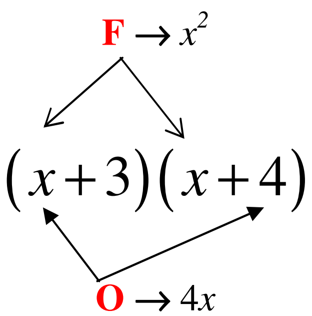
Now look at the parts of the garden which make up: $x^2+4x+3x+12$

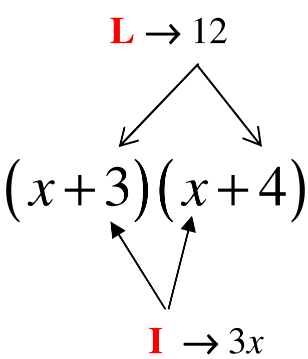
The x^2 term comes from the x in each bracket – the **First** term

The $4x$ term comes from the x in the first bracket and the 4 in the second bracket – the **Outer** terms

The $3x$ term comes from the 3 in the first bracket and the x in the second bracket – the **Inner** terms

The 12 term comes from the 3 in the first bracket and the 4 in the second bracket – the **Last** terms





FOIL
is a useful way of remembering the order in which to break a pair of brackets

Breaking Pairs of Brackets – FOIL ... *continued*

This method can be used in many different cases:

e.g. $(2x-1)(x+3) \rightarrow 2x^2+6x-x-3 \rightarrow 2x^2+5x-3$

How do we deal with this $(x+3)^2$?

$(x+3)^2$ – if we think about what it means, then **squared** just means multiplied by itself.

So, $(x+3)^2 \rightarrow (x+3)(x+3)$
and we know how to break this pair of brackets using FOIL.

Hence, $(x+3)^2 \rightarrow (x+3)(x+3) \rightarrow x^2+3x+3x+9 \rightarrow x^2+6x+9$

Two useful squares to be aware of are:

$(a+b)^2 \rightarrow (a+b)(a+b) \rightarrow a^2+ab+ab+b^2 \rightarrow a^2+2ab+b^2$

$(a-b)^2 \rightarrow (a-b)(a-b) \rightarrow a^2-ab-ab+b^2 \rightarrow a^2-2ab+b^2$

Equations with brackets

Finally, we can extend this to equations with pairs of brackets:

e.g. $(y-3)^2 = y(y+3)$

$(y-3)(y-3) = y(y+3)$ *(write out the square in full)*

$y^2-3y-3y+9 = y^2+3y$ *(Use FOIL on the Left hand side, and break the brackets on the right)*

$y^2-6y+9 = y^2+3y$ *(simplify)*

$-6y+9 = 3y$ *(subtract y^2 from both sides)*

$9 = 9y$ *(add 6y to both sides)*

$1 = y$ *(Divide both sides by 9)*

SUMMARY:

1. Use of Signs – **Same Sign Plus**
2. **Simplify** wherever possible
3. Use **FOIL** to break a pair of brackets
4. Noting that **squared** – means multiplied by itself.
5. Remember that $2m \times 2m \rightarrow 2\times m\times 2\times m \rightarrow 4m^2$
6. Two useful squares $(a+b)^2 \rightarrow a^2+2ab+b^2$
   $(a-b)^2 \rightarrow a^2-2ab+b^2$