The Straight Line

Gradient:

The gradient of a straight line is: $\frac{\text{Rise}}{\text{Run}}$ and

The gradient is usually denoted by m

y–intercept:

The y–intercept of a straight line where the line crosses the y–axis.

The y–intercept is usually denoted by c

Equation of a straight line:

The equation of a straight line is given by:

$$y = mx + c$$

where m is the gradient and c is the y–intercept.

Finding the equation of a line.

We need the gradient m and the y–intercept c.

Example:

What is the equation of the straight line with gradient 2 and y–intercept 3 ?

Solution:

m = 2, c = 3 Hence, since $y = mx + c$ then, $y = 2x + 3$

Two Points:

We may be given two points on a graph, one of which is the y–intercept.

From the two points we can work out the gradient – so we have m.

If one point is the y–intercept then we also have c.

So we can now write down the equation, substituting for m and c in $y = mx + c$

Using the Equation of the Straight Line.

If we are given the equation of a line, then we can find out where it cuts the x and y axes.

y–intercept:

If we put $x = 0$ into the equation, we can find where the line cuts the y–axis.

e.g. $y = 3x - 1$ putting $x = 0$ gives $y = 3(0) - 1$ $y = -1$

so, y–intercept = –1

x–intercept:

If we put $y = 0$ into the equation, we can find where the line cuts the x–axis.

e.g. $y = 2x - 4$ putting $y = 0$ gives $0 = 2x - 4$ $\rightarrow$ $2x = 4$ $\rightarrow$ $x = 2$

so, x–intercept = 2

A point lies on a line, if it satisfies the equation of the line.

If we substitute the coordinates of the point into the equation of the line, then the equation will be true i.e. satisfied.

e.g. Does the point (1, 2) lie on the line $y = 3x - 1$

Substitute into the equation – is it true?

$2 = 3(1) - 1$ $\rightarrow$ $2 = 3 - 1$ $\rightarrow$ $2 = 2$ this is true, so point does lie on the line.

e.g. Does the point (3, 4) lie on the line $y = 3x - 1$

Substitute into the equation – is it true?

$4 = 3(3) - 1$ $\rightarrow$ $4 = 9 - 1$ $\rightarrow$ $4 = 8$ this is NOT true,

so point does NOT lie on the line.

On the next page are some past paper questions.

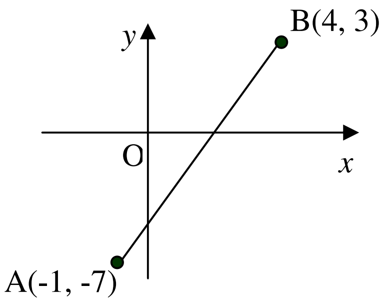
Past Paper Questions:

1. In the diagram, A is the point (-1, 7) and B is the point (4, 3).

   a) Find the gradient of the line AB.

   b) AB cuts the y-axis at the point (0, -5). Write down the equation of the line AB

   c) The point (3*k*, *k*) lies on AB. Find the value of *k*.



Solution

1. a) Gradient AB = $\dfrac{3-(-7)}{4-(-1)} \rightarrow \dfrac{10}{5} \rightarrow 2$

   b) Use y = mx + c Eqn is: $y = 2x - 5$

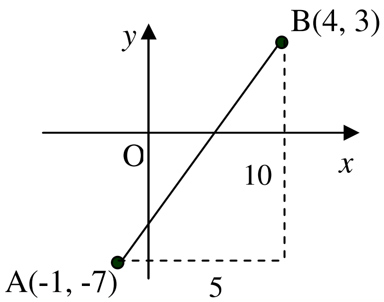
   c) (3k, k) lies on AB, so it will satisfy the equation

   Hence, $k = 2(3k) - 5$

   $k = 6k - 5$

   $5 = 5k$

   $k = 1$

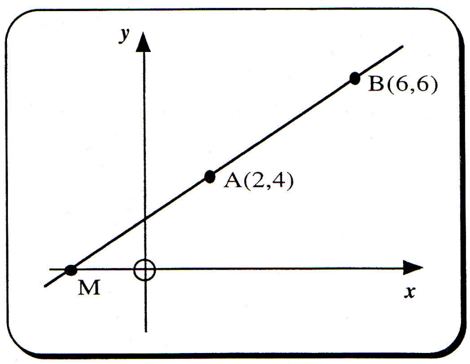


2. The straight line through the points A(2, 4) and B(6, 6) is shown in the diagram.

   The point M is where the line AB cuts the x-axis.

   a) Find the equation of the straight line AB.

   b) Use this equation to find the coordinates of the point M.



Solution

2. a) Gradient AB = $\dfrac{6-4}{6-2} \rightarrow \dfrac{2}{4} \rightarrow \dfrac{1}{2}$

   Use y = mx + c, so $y = \frac{1}{2}x + c$

   Need to find c, so use point (2, 4)

   $4 = \frac{1}{2}(2) + c \quad 4 = 1 + c \quad c = 3$

   Equation is $y = \frac{1}{2}x + 3$

   b) To find M, we know that $y = 0$

   Hence $0 = \frac{1}{2}x + 3$ solving gives $x = -6$