Trig Equations

Angles greater than 90°

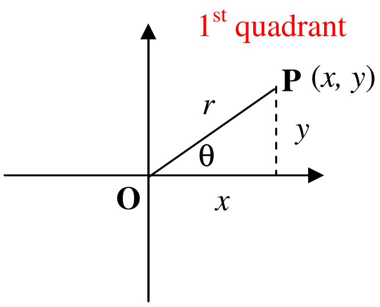
We need to re–define sin, cos and tan to be able to deal with angles greater than 90°.

Instead of defining them in terms of the opposite, adjacent and hypotenuse, we can use the coordinates of a rotating line.

THEORY

A rotating line:

Imagine a line OP of length, *r*, rotating about the origin in an anti–clockwise direction starting on the *x*–axis

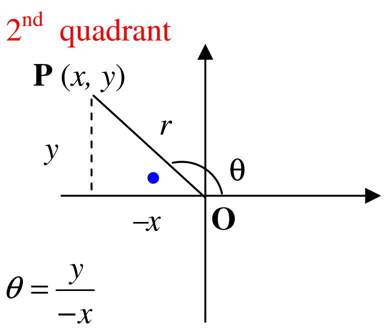


At any point, the coordinates of P are (x, y) and the angle between the line OP and the positive direction of the x–axis is denoted as θ

In the 1st quadrant we can now define: $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$

Notice that all **3 ratios** are **POSITIVE** in the 1st quadrant.

Now let the line OP move into the 2nd quadrant.

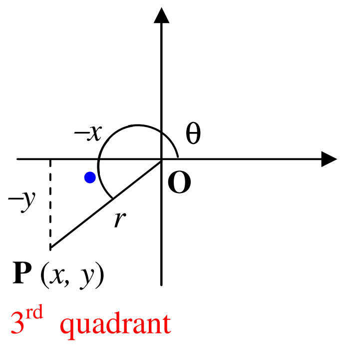


We define $\sin\theta$, $\cos\theta$ and $\tan\theta$ based on the acute angle between the line OP and the x–axis.
(angle marked with •)

So in the 2nd quadrant we have: $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{-x}{r}$, $\tan\theta = \frac{y}{-x}$

Notice that only the **sine ratio** is **POSITIVE** in the 2nd quadrant.

Now let the line OP move into the 3rd quadrant.



We define $\sin\theta$, $\cos\theta$ and $\tan\theta$ based on the acute angle between the line OP and the x–axis.
(angle marked with •)

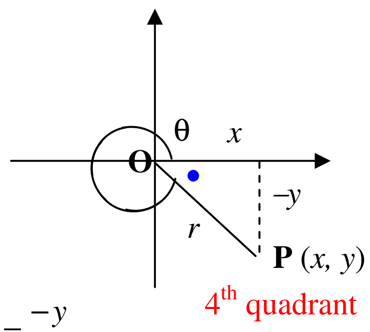
So in the 3rd quadrant we have: $\sin\theta = \frac{-y}{r}$, $\cos\theta = \frac{-x}{r}$, $\tan\theta = \frac{-y}{-x}$

Notice that only the **tangent ratio** is **POSITIVE** in the 3rd quadrant.

THEORY continued:

Now let the line OP move into the 4th quadrant.

We define $\sin\theta$, $\cos\theta$ and $\tan\theta$ based on the acute angle between the line OP and the x–axis. (angle marked with •)



So in the 4th quadrant we have: $\sin\theta = \frac{-y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{-y}{x}$

Notice that only the **cosine ratio** is **POSITIVE** in the 4th quadrant.

Summary:

For angles greater than 90°, represented by a line OP the sine, cosine and tangent are defined as the sine, cosine and tangent of the acute angle between OP and the x–axis.

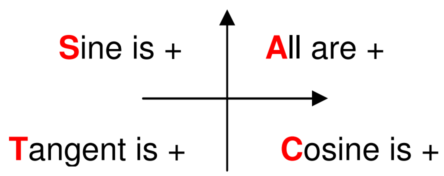
The sign is either positive or negative according to whether the sine, cosine and tangent is positive or negative in that quadrant.

From the above theory:

1st Quadrant	All were positive
2nd Quadrant	Sine was positive
3rd Quadrant	Tangent was positive
4th Quadrant	Cosine was positive

There is an easy way to remember the signs. **A**ll **S**inners **T**ake **C**are

A quick sketch explains this.



METHOD:

For angles greater than 90°, draw a 4 quadrant axis, and mark on where the angle is.

Mark in the acute angle and calculate it by addition or subtraction using 180° or 360° as appropriate.

Take the sign from: **A**ll **S**inners **T**ake **C**are

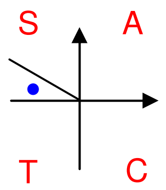
Some examples will make this clear.

e.g. sin 150°

acute angle is marked •
and is 180 – 150 = 30°

sin is + in 2nd quadrant

So sin 150° = sin 30°

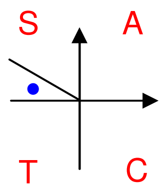


e.g. tan 150°

acute angle is marked •
and is 180 – 150 = 30°

tan is – in 2nd quadrant

So tan 150° = –tan 30°

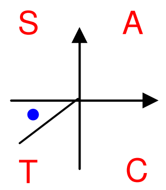


e.g. cos 220°

acute angle is marked •
and is 220 – 180 = 40°

cos is – in 3rd quadrant

So cos 220° = –cos 40°



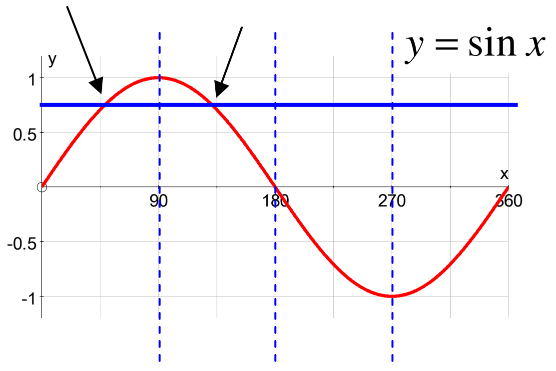
The main application for this result is when we are working back to the angle.

E.g. If $\sin\theta = 0.707$ what is θ

We should look back at the graph at this point and consider where the graph is 0.707

There are two values of θ where $\sin\theta = 0.707$
In the 1st and 2nd quadrants.

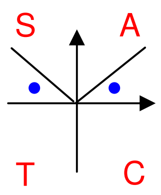
This should not be a surprise, as we know that the sign is + in these quadrants.



So, if $\sin\theta = 0.707$ then the acute value of θ is given by: $\theta = \sin^{-1}(0.707)$ $\theta = 45°$

Since $\sin\theta = +0.707$, then we have values in 1st and 2nd quadrants

Use ASTC



So our two angles are: 45° and 180 – 45 = 135°

This all seems quite complicated, so we need to simplify it all.

We will use some simple rules.

We will be starting with $\sin\theta = ...,\quad \cos\theta = ...$ or $\tan\theta = ...$

e.g. $\sin\theta = -0.35$ or $\cos\theta = 0.93$ etc.

Method:

1. Ignore the sign
2. Use the inverse key on your calculator

   i.e. $\sin^{-1}(....),\ \cos^{-1}(....)$ or $\tan^{-1}(....)$

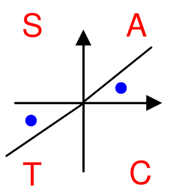
   This will give you the acute angle.
3. Now look at the sign, and use ASTC
   to determine which 2 quadrants the angles must be in
4. Mark in the acute angles on your diagram.
5. Calculate the actual angles

Example:

$\tan\theta = 0.56$ Find the possible values of θ

(1) Ignore the sign **(2)** find the acute angle $\theta = \tan^{-1}0.56 \rightarrow \theta = 29.2°$ – Round to 29°

(3) tan is +, now use ASTC



(4) Tangent is positive in 1st and 3rd quadrants.

(5) Acute angle is 29° so actual angles are:

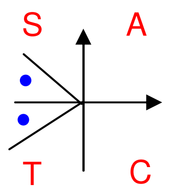
29° *(1st quadrant)* and 180 + 29 = 209° *(3rd quadrant)*

Example:

$\cos\theta = -0.23$ Find the possible values of θ

(1) Ignore the sign **(2)** find the acute angle $\theta = \cos^{-1}0.23 \rightarrow \theta = 76.7°$ – Round to 77°

(3) cos is –, now use ASTC



(4) Cosine is negative in 2nd and 3rd quadrants.

(5) Acute angle is 77° so actual angles are:

180 – 77 = 103° *(2nd quadrant)*

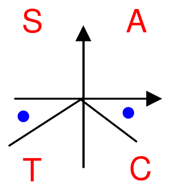
and 180 + 77 = 257° *(3rd quadrant)*

Example:

$\sin\theta = -0.88$ Find the possible values of θ

(1) Ignore the sign **(2)** find the acute angle $\theta = \sin^{-1} 0.88 \rightarrow \theta = 61.6°$ – Round to 62°

(3) sin is –, now use ASTC



(4) Sine is negative in 3rd and 4th quadrants.

(5) Acute angle is 62° so actual angles are:

180 + 62 = 242° *(3rd quadrant)*

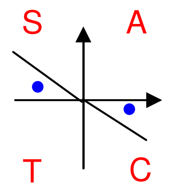
and 360 – 77 = 23° *(4th quadrant)*

Example:

$\tan\theta = -0.32$ Find the possible values of θ

(1) Ignore the sign **(2)** find the acute angle $\theta = \tan^{-1} 0.32 \rightarrow \theta = 17.7°$ – Round to 18°

(3) tan is –, now use ASTC



(4) Tangent is negative in 2nd and 4th quadrants.

(5) Acute angle is 18° so actual angles are:

180 – 18 = 162° *(2nd quadrant)*

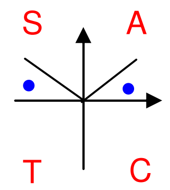
and 360 – 18 = 342° *(4th quadrant)*

Example:

$\sin\theta = 0.5$ Find the possible values of θ

(1) Ignore the sign **(2)** find the acute angle $\theta = \sin^{-1} 0.5 \rightarrow \theta = 30°$

(3) sin is +, now use ASTC



(4) Sine is positive in 1st and 2nd quadrants.

(5) Acute angle is 30° so actual angles are:

30° *(1st quadrant)*

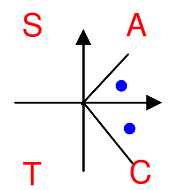
and 180 – 30 = 150° *(2nd quadrant)*

Example:

$\cos\theta = 0.75$ Find the possible values of θ

(1) Ignore the sign **(2)** find the acute angle $\theta = \cos^{-1} 0.75 \rightarrow \theta = 41.4°$ – Round to 41°

(3) cos is +, now use ASTC



(4) Cosine is positive in 1st and 4th quadrants.

(5) Acute angle is 41° so actual angles are:

41° *(1st quadrant)*

and 360 – 41 = 319° *(4th quadrant)*

Applications:

The applications of this at Standard Grade are in solving simple trig equations.

We will be given an equation, which has to be re–arranged to the form:

$$\sin\theta = ..., \quad \cos\theta = ... \quad \text{or} \quad \tan\theta = ...$$

once we have done this, then we simply use ASTC as above to find the 2 angles.

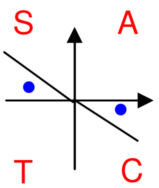
Example:

Solve the equation $3\tan x^o + 5 = 0$, for $0 \le x \le 360$.

Re–arrange the equation: $3\tan x^o + 5 = 0 \rightarrow 3\tan x^o = -5 \rightarrow \boxed{\tan x^o = -\frac{5}{3}}$

(1) Ignore the sign **(2)** find the acute angle $x = \tan^{-1}\left(\frac{5}{3}\right) \rightarrow x = 59.03°$ – Round to 59°

(3) tan is –, now use ASTC



(4) Tangent is negative in 2nd and 4th quadrants.

(5) Acute angle is 59° so actual angles are:

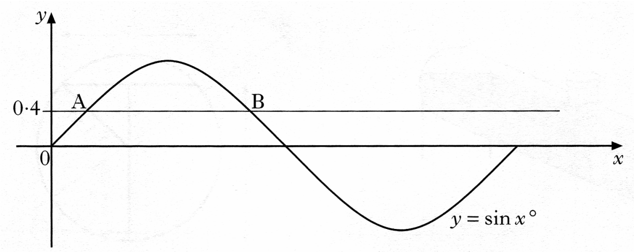
180 – 59 = 121° *(2nd quadrant)*

and 360 – 59 = 301° *(4th quadrant)*

Hence: x = 121° and x = 301°

Example:

The diagram shows part of the graph of $y = \sin x$.



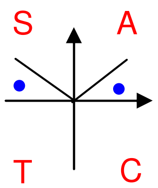
The line $y = 0.4$ is drawn and cuts the graph of $y = \sin x$ at A and B.

Find the x-coordinates of A and B.

We have to solve: $\sin x = 0.4$

(1) Ignore the sign **(2)** find the acute angle $x = \sin^{-1} 0.4 \rightarrow x = 23.6°$ – Round to 24°

(3) sin is +, now use ASTC



(4) Sine is positive in 1st and 2nd quadrants.

(5) Acute angle is 24° so actual angles are:

24° *(1st quadrant)*

and 180 – 24 = 156° *(2nd quadrant)* $x_A = 24°$ $x_B = 156°$

Past Paper Questions:

1. Solve **algebraically** the equation $2+3\sin x° = 0$ for $0 \le x \le 360$

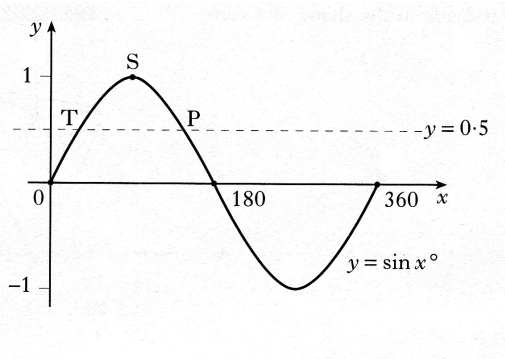
2. Solve **algebraically**, the equation $7\cos x° - 2 = 0$ for $0 \le x \le 360$

3. Solve **algebraically**, the equation $5\tan x - 9 = 0$, for $0 \le x \le 360$

4. Solve the equation $5\sin x° + 2 = 0,\ for\ 0 \le x \le 360$

5. Solve algebraically the equation: $\tan 40° = 2\sin x° + 1$ $0 \le x \le 360$

6. The diagram shows the graph of $y = \sin x°$, $0 \le x \le 360$

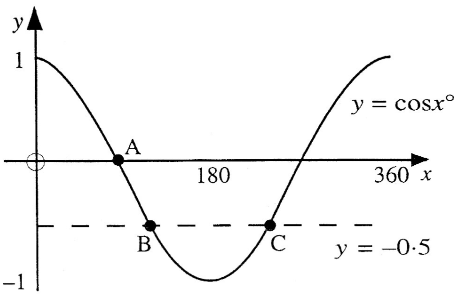


a) Write down the coordinates of point S.

The straight line $y = 0.5$ cuts the graph at T and P.

b) Find the coordinates of T and P.

7. The diagram shows the graph of $y = \cos x°$, $0 \le x \le 360$.



a) Write down the coordinates of point A.

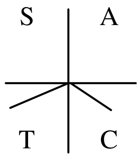
The straight line $y = -0.5$ cuts the graph at B and C.

b) Find the coordinates of B and C.

Solutions follow on the next page.

Solutions:

1. $2+3\sin x = 0 \quad \rightarrow \quad \sin x = -\frac{2}{3}$



$x = \sin^{-1}\left(\frac{2}{3}\right) \qquad \text{acute } x = 41.81..°$

$x = 180+42 = 222° \quad or \quad x = 360-42 = 318°$

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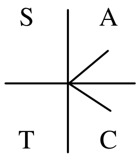
2. $7\cos x - 2 = 0 \quad \rightarrow \quad \cos x = \frac{2}{7}$



$x = \cos^{-1}\frac{2}{7} \qquad \text{acute } x = 73.398..°$

$x = 73° \quad or \quad x = 360-73 = 287°$

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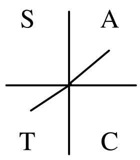
3. $5\tan x - 9 = 0 \quad \rightarrow \quad \tan x = \frac{9}{5}$



$x = \tan^{-1}\frac{9}{5} \qquad \text{acute } x = 60.945..°$

$x = 61° \quad or \quad x = 180+61 = 241°$

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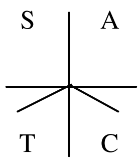
4. $5\sin x + 2 = 0 \quad \rightarrow \quad \sin x = -\frac{2}{5}$



$x = \sin^{-1}\frac{2}{5} \qquad \text{acute } x = 23.578..°$

$x = 180+24 = 204° \quad or \quad x = 360-24 = 336°$

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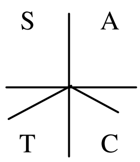
5. $\tan 40 = 2\sin x + 1 \quad \rightarrow \quad \sin x = -\frac{0.1609}{2}$



$x = \sin^{-1}\frac{0.1609}{2} \qquad \text{acute } x = 4.614..°$

$x = 180+5 = 185° \quad or \quad x = 360-5 = 355°$

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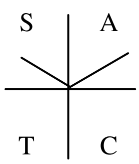
6. a) S is (90°, 1)

   b) $\sin x = 0.5$



$x = \sin^{-1} 0.5 \qquad \text{acute } x = 30°$

$x = 30° \quad or \quad x = 180-30 = 150°$

T is (30°, 0.5) and P is (150°, 0.5)

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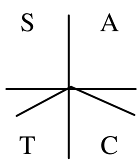
7. a) A is (90°, 0)

   b) $\cos x = -0.5$



$x = \cos^{-1} 0.5 \qquad \text{acute } x = 60°$

$x = 180+60 = 240° \quad or \quad x = 360-60 = 300°$

B is (240°, -0.5) and C is (300°, -0.5)
